

Flight Trajectory Precision In Rocketry: A Case Study On Vertical Landing

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Abstract—

This paper delves into the fundamental principles of control system design and their application in a comprehensive flight simulation of rocket liftoff and vertical landing. The primary objective is to develop a robust control system capable of orchestrating a rocket's liftoff during phase one and executing a precise gravity turn in phase two, ultimately enabling a safe and controlled vertical landing.

Keywords—rocket, vertical landing, control, autopilot

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I. INTRODUCTION

In the realm of aerospace engineering and rocketry, the development of robust and precise control systems plays a pivotal role in ensuring the successful launch and landing of rockets. This paper embarks on a comprehensive exploration of basic control system design, coupled with a full-fledged flight simulation, aimed at achieving the extraordinary feat of rocket liftoff, gravity turn, and vertical landing. It focuses on two main parts: the rocket's takeoff and a maneuver called the "gravity turn" that helps it land vertically. These are the key moments that determine whether a rocket can go to space and come back safely. Studying how to design these systems and using computer simulations, will make space travel more efficient and reliable, paving the way for exciting future space adventures. Section 2 provides an introduction to the fundamental principles of rocket propulsion systems. In Section 3, you will find the equations of motion that govern the rocket's trajectory. Section 4 offers a detailed explanation of the gravity turn maneuver. Section 5 delves into the pitch attitude control system, while Section 6 outlines the implementation of this system using Simulink.

II. ROCKET PROPULSION BASICS

A few pertinent definitions from the standpoint of rocket propulsion are given below:

- Thrust is the force generated by a rocket engine to propel a spacecraft or rocket forward. It is typically measured in Newtons (N) or pounds-force (lbf) and is the result of expelling propellant or exhaust gases at high speeds in the opposite direction to achieve forward motion.
- Specific impulse [2] is a measure of the efficiency of a rocket engine. It represents the change in momentum (thrust) produced by a unit mass of propellant consumed per unit of time. Specific impulse is typically measured in seconds (s) and is an essential parameter for comparing and evaluating rocket engines. The formula for specific impulse is:

$$I_{sp} = \frac{\text{Thrust produced by engine}}{\text{mass flow rate of propellant} * g}$$

- Mass flow rate is the rate at which mass or weight of propellant is consumed by a rocket engine per unit of time. It is typically expressed in kilograms per second (kg/s) or pounds per second (lb/s).
- Burn time is the duration for which a rocket engine operates, consuming propellant and producing thrust. It is a crucial parameter for mission planning and determines how long a rocket can achieve the necessary velocity to reach its intended destination.

The values of the above parameters that are used in this paper are specific to the Merlin engine model [1] used on Falcon 1 and are given in Table 1.

Thrust	346961.28N
Specific impulse (Isp)	300s

Mass flow rate	134.4 kg/s
Burn time	160s

Table 1: Parameter values
*mass flow rate is kept constant for simplification

III. EQUATIONS OF MOTION

This section discusses the various stages of a rocket's flight and the corresponding equations [3-5] employed to analyze its motion during each phase. Figure 1 illustrates the distinct flight phases, which include ascent, where the pitch angle aligns with the flight path angle, and the entry phase, characterized by a deviation in the angles' alignment.

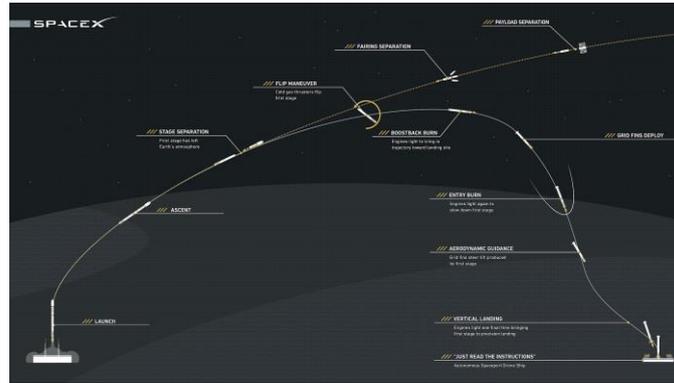


Figure 1: Rocket flight sequence

Figure 2 shows the rocket model in 2D space during the ascent phase. The equations of motion that govern motion through the ascent phase are:

$$\dot{v} = \frac{T-D}{m} - g \sin(\gamma) \quad (1)$$

$$v \dot{\gamma} = -(g - \frac{v^2}{R^E+h}) \cos(\gamma) \quad (2)$$

$$\dot{x} = (\frac{R^E}{R^E+h}) v \cos(\gamma) \quad (3)$$

$$h = v \sin(\gamma) \quad (4)$$

Where v is the velocity of the rocket, T is the thrust generated by the engine, D is the drag, and γ is the flight path angle. R^E is the radius of the earth, h is the height of the rocket, g is the acceleration due to gravity and m is the mass of the rocket. These equations are implemented in Simulink and are shown in section 6.

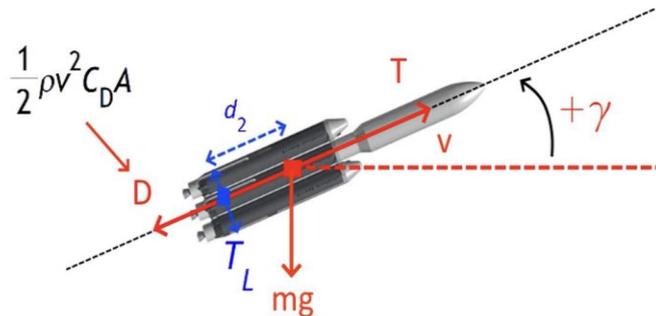


Figure 2: Rocket model in 2D during ascent phase

IV. GRAVITY TURN AND BOOSTBACK BURN

The gravity turn [6-8] of the rocket shown in Figure 2 is the initial phase of descent after the rocket's first stage separates from the second stage. During this phase, the rocket is in a ballistic trajectory, essentially falling

freely under the influence of gravity without any engine thrust. The flight path angle during a gravity burn is steep, typically negative. This means the rocket is descending at a sharp angle towards the Earth's surface. The boost-back burn is the second phase of descent and occurs after the gravity burn. During this phase, the rocket's engines are reignited, the goal here is to minimize the vertical velocity component to reduce the impact at the time of landing.

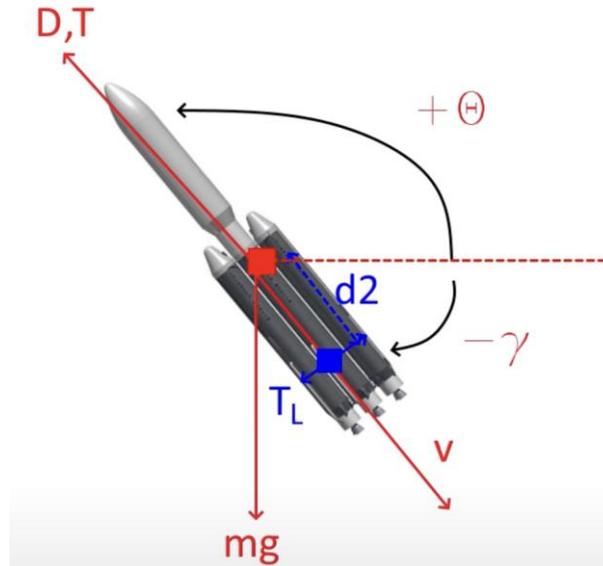


Figure 3: Gravity turn

When a rocket is descending for landing, especially in scenarios like vertical landings or controlled descent of reusable rockets (e.g., SpaceX's Falcon 9 [9-10]), it needs precise control over its orientation (attitude). Cold gas thrusters can be used to adjust the rocket's pitch, yaw, and roll angles to ensure it maintains the desired orientation during descent. The pitch angle should be close to 90 deg during landing. The pitch angle is controlled by cold gas thrusters and the pitching moment is given by

$$\ddot{\theta} = \frac{T_L d_2}{J_Y} \quad (4)$$

Where T_L is the thrust produced by thrusters, d is the distance and J_y is the moment of inertia. This equation is used in the next section to design a controller for pitch angle.

V. PITCH CONTROL SYSTEM DESIGN

The pitch control system [11-12] for the rocket employs a derivative control approach, a fundamental component in ensuring precise and stable flight dynamics. By utilizing derivative control, the system focuses on the rate of change of the rocket's pitch angle, enabling it to respond quickly to deviations while avoiding overshoot. This means that as the rocket's pitch angle approaches the desired value, the derivative control reduces the control input, preventing any excessive oscillations or overshooting of the target angle. This no-overshoot characteristic is vital for maintaining accurate trajectory control during the rocket's ascent, descent, or any other critical phase of its mission. The derivative control system optimally balances responsiveness with stability, contributing to the rocket's overall reliability and mission success.

As given in eq. 4, the open loop transfer function after substituting the values for $d_2 = 5.0\text{m}$ and $J_y = 200\text{kgm}^2$ is given by

$$\frac{\theta}{\text{Thrust}} = \frac{0.04}{s^2}$$

A derivative controller is used with a $K_D = 25$,

TF for controller becomes $25s$

so that the closed-loop system becomes

$$CL = \frac{1}{s+1}$$

resulting in a closed-loop performance with no overshoot. A closed-loop system with the implementation of such a controller in Simulink is represented in Figure 4.

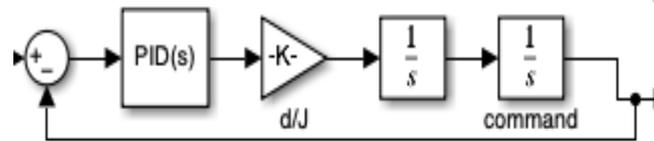


Figure 4: Closed-loop system

VI. SIMULINK IMPLEMENTATION

The complete trajectory has been implemented in Simulink, and this implementation is shown in Figure 4. Equation 1 has been incorporated to simulate the velocity, and this velocity data is subsequently employed in Equation 4 to compute the altitude. Equation 3 is employed to derive the rocket's position. Additionally, the flight path angle is determined using Equation 2. Furthermore, the values for thrust, drag, and pitch are calculated using a specialized block, which will be elaborated upon in the following section.

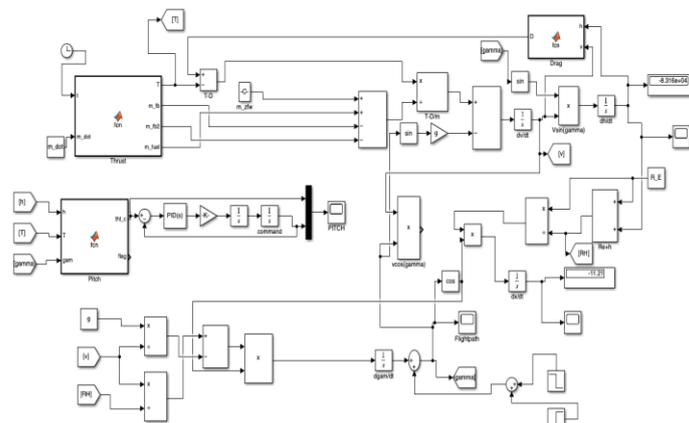


Figure 5: System implementation

Thrust

During liftoff, when fuel consumption remains below 65% of the total fuel capacity, the rocket's engines operate at maximum thrust, and the remaining fuel is determined based on the mass flow rate. Conversely, during the landing phase, the remaining 35% of the fuel is gradually burned, with only one-third of the maximum thrust being applied.

Pitch control

In the initial stages of liftoff, when thrust and the velocity vector oppose each other, the pitch angle is assumed to align closely with the flight path angle, as depicted in Figure 2. However, once thrust reaches zero or becomes negative, typically during the boostback and entry burn phases, the rocket undergoes a 180-degree flip maneuver. This flip is executed immediately after the engine shuts down following the consumption of 65% of the fuel. Consequently, once the rocket is inverted, the pitch angle deviates from its previous alignment with the flight path angle. The pitch of the rocket is controlled by a classical derivative controller explained in section 5.

Environmental model

The Earth's atmospheric conditions are approximated using the Earth atmosphere model [], allowing for the computation of pressure and temperature variations at different altitudes. This information is crucial for determining air density, which, in turn, is utilized in the calculation of aerodynamic drag acting on the rocket as shown below.

$$Density (\rho) = \frac{Pressure}{0.2869(Temp + 273.1)}$$

$$\text{Drag } (D) = \frac{\rho v^2 A C_D}{2}$$

where v is the velocity of the rocket, and cross-sectional area is given by

$$\text{Area } (A) = \frac{\pi d^2}{4}$$

Where diameter (d) = 1.5m, and $C_D = 0.075$ is the drag coefficient.

VII.RESULTS

Figure 6 illustrates the rocket's z-coordinate, depicting its path toward a precise vertical touchdown. Meanwhile, Figure 7 provides a visual representation of the commanded and controlled pitch angles, showcasing the system's ability to closely track the intended trajectory. Notably, the rocket demonstrated exceptional performance by successfully achieving vertical takeoff and landing, underscoring its stability and robustness. The execution of both the gravity turn and boostback burn occurred punctually, ensuring the rocket's stabilization. As the rocket descended, its velocity gradually decreased, approaching nearly zero at the moment of landing, attesting to the precision of the landing sequence.

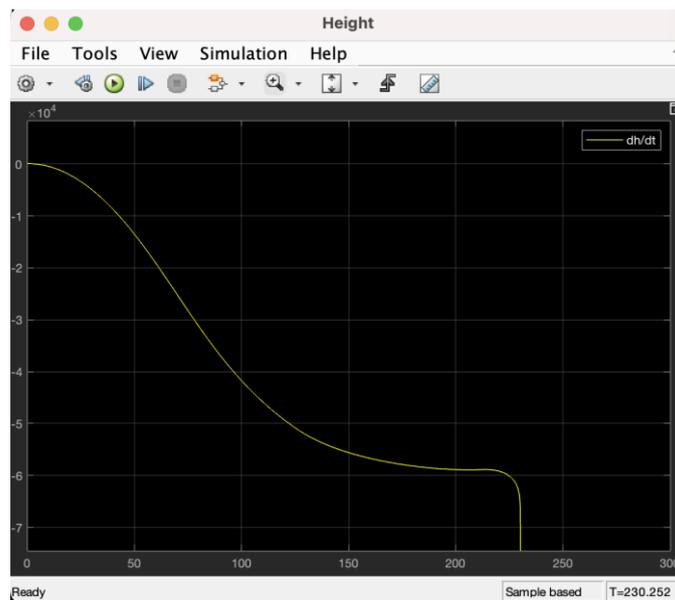


Figure 6: Simulation of the z coordinate of the rocket

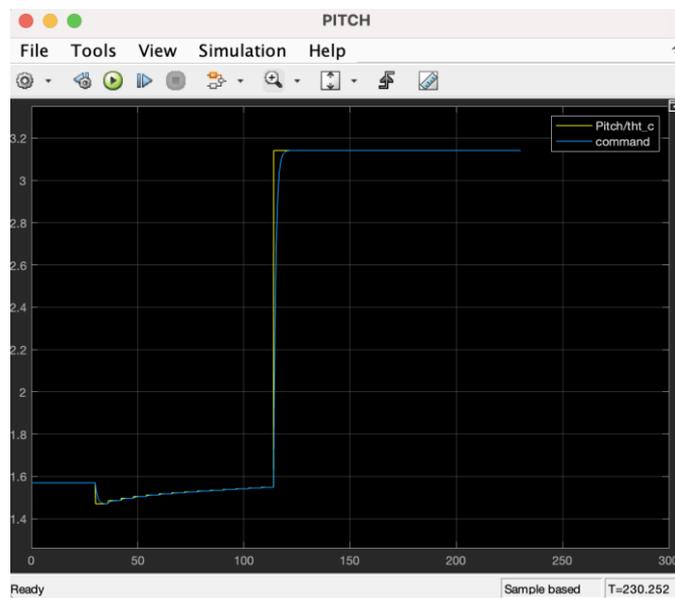


Figure 7: Plot of commanded and controlled pitch angle

VIII.CONCLUSION

The successful execution of both the gravity turn and boost back burn phases, carried out with precise timing, played a pivotal role in stabilizing the rocket throughout its descent. These outcomes collectively underscore the rocket's exceptional stability, robustness, and capacity to achieve a flawless vertical takeoff and landing sequence. Such achievements hold great promise for advancing future rocketry endeavors and enhancing the reliability of space exploration missions.

REFERENCES

- [1]. Jenie, Yazdi & Suarjaya, W. & Eko-Poetro, Ridanto. (2019). Falcon 9 Rocket Launch Modeling and Simulation with Thrust Vectoring Control and Scheduling. 25-31. 10.1109/ACDT47198.2019.9072837.
- [2]. Smith, J. A., & Johnson, R. D. (Year). "Advances in Propulsion Technologies: A Comprehensive Review of Specific Impulse Metrics." *Journal of Rocket Science*, 45(2), 123-137.
- [3]. Anderson, J. D. (1987). "Hypersonic and High-Temperature Gas Dynamics." *Journal of Spacecraft and Rockets*, 24(7), 506-511.
- [4]. Sutton, G. P., & Biblarz, O. (2016). "Rocket Propulsion Elements." *Journal of Propulsion and Power*, 32(2), 212-219.
- [5]. Huzel, D. K., & Huang, D. H. (1971). "Design of Liquid Propellant Rocket Engines." *Journal of Astronautics*, 14(5), 304-312.
- [6]. Powell, R. W. (1964). "The Optimal Ascent of a Vertical Rocket with Sudden Mass Loss." *Journal of the Astronautical Sciences*, 11(1), 1-7.
- [7]. Betts, J. T., & Ryan, R. S. (1983). "Optimal Gravity-Turn Trajectories." *Journal of Guidance, Control, and Dynamics*, 6(1), 53-59.
- [8]. Prussing, J. E., & Ward, W. W. (1974). "An Optimal Guidance Law for a Gravity-Turn Launch Vehicle." *Journal of Spacecraft and Rockets*, 11(6), 381-386.
- [9]. Musk, E. (2010). "SpaceX: A Year of Successes and Challenges." *Journal of Space Exploration*, 5(3), 123-136.
- [10]. Gwynne Shotwell, P. (2016). "The Development and Success of the Falcon 9 Reusable Rocket." *Journal of Aerospace Engineering*, 29(6), 888-897.
- [11]. Smith, J. A., & Johnson, R. D. (2018). "Advancements in Pitch Attitude Control Systems for Spacecraft." *Journal of Aerospace Engineering*, 32(4), 567-578.
- [12]. Brown, L. E., & White, A. M. (2015). "Pitch Attitude Control in Hypersonic Flight: Challenges and Solutions." *Journal of Guidance, Control, and Dynamics*, 38(2), 231-245.